1 Formal Hopcroft & Ullman (1979, p. 148) description of a Turing Machine that plays the Bad Apple!! PV

The following described Turing Machine M plays the **Bad Apple!! PV** on an infinite tape using an enumerator and a backtracking state. A binary tree within the transition function which handles the enumerator has a sequence of states appended to each leaf that refreshes the cells associated with the display to the corresponding frame every enumeration.

Given a total length Δ, width W, height H, and function B,

$$B : ([0, \Delta), [0, W), [0, H)) \to \{0, 1\}$$

where $B(p, x, y)$ represents the $(yH + x)$th pixel of frame p,

Let $w = \lceil \log_2 \Delta \rceil$ represent the bit-length of Δ.

Let $N = 2^{w+1} + WH\Delta + 7$ represent the total number of non-final states.

Let $M = \langle Q, \Gamma, b, \Sigma, \delta, q_0, F \rangle$ represent a Turing Machine where

$Q = \{q_0, q_1, q_2, q_3, ...q_N, q_{\text{accept}}, q_{\text{reject}}\}$,
$\Gamma = \{0, 1, \sqcup, \#\}$,
$b = \sqcup \in \Gamma$,
$\Sigma = \{0, 1\} \subseteq \Gamma \setminus \{b\}$,
$F = \{q_{\text{accept}}, q_{\text{reject}}\}$
with transition function δ defined as

$\delta(q_i, i) = (q_{\text{reject}}, i, R)$ for all $i \in [0, N]$, $i \in \Gamma$ except in all following cases:

$$\delta(q_0, 0) = (q_1, \sqcup, R),$$
$$\delta(q_0, 1) = (q_0, \sqcup, R),$$
$$\delta(q_1, 0) = (q_1, 0, R),$$
$$\delta(q_1, 1) = (q_2, \#, R),$$

$$\begin{aligned}
\delta(q_2, 0) &= (q_2, 0, R), \\
\delta(q_2, \sqcup) &= (q_3, \#, L), \\
\delta(q_3, 0) &= (q_3, 0, L), \\
\delta(q_3, 1) &= (q_3, 1, L), \\
\delta(q_3, \#) &= (q_3, \#, L), \\
\delta(q_3, \sqcup) &= (q_4, \sqcup, R).
\end{aligned}$$

Let $d = 2^{w+1}$ represent the number of states in the enumerator. For $j \in [1, d]$,

Let $\alpha = 2j$.

Let $\beta = 1 + d + (j - 2^w)WH$.

$$\begin{aligned}
\delta(q_{3+j}, 0) &= (q_{3+\alpha}, 0, R), \quad \alpha \leq d, \\
\delta(q_{3+j}, 1) &= (q_{4+\alpha}, 1, R), \quad \alpha + 1 \leq d, \\
\delta(q_{3+j}, \#) &= (q_{3+\beta}, \#, R), \quad \alpha > d.
\end{aligned}$$

If $\alpha > d$, for $k \in [3 + \beta, 3 + \beta + WH)$,

Let $p = j - 2^w$.

Let $y = \lfloor \frac{k - \beta - 3}{H} \rfloor$ and $x = (k - \beta - 3)\text{mod}H$.

$$\delta(q_k, i) = (q_{k+1}, B(p, x, y), R) \text{ for } i \in \{0, 1\}$$

Let $q_{\text{back}} = q_{N-2}$ represent the backtracking state.

Let $q_{\text{inc}} = q_{N-1}$ represent the incrementing state.

$$\delta(q_{(\beta+WH)}, \#) = (q_{\text{back}}, \#, L).$$

Define the following for q_{back} and q_{inc}:

$$\begin{aligned}
\delta(q_{\text{back}}, 0) &= (q_{\text{back}}, 0, L), \\
\delta(q_{\text{back}}, 1) &= (q_{\text{back}}, 1, L), \\
\delta(q_{\text{back}}, \#) &= (q_{\text{inc}}, \#, L), \\
\delta(q_{\text{inc}}, 0) &= (q_3, 1, L), \\
\delta(q_{\text{inc}}, 1) &= (q_{\text{inc}}, 0, L),
\end{aligned}$$

and with the transition to the final accept state:

$$\delta(q_N, \#) = (q_{\text{accept}}, \#, R)$$

Run M on input $1^{W-w-2}0^{w+1}10^{WH}$ and view the tape as a matrix of width W.